تمرین شماره یک

① سری فوریه تابع $f(x)=x$ را در بازه $(0,2\pi)$ بیابید.

$$T = 2\pi - 0 = 2\pi$$

$$a_0 = \frac{1}{T}\int f(x)\,dx = \frac{1}{2\pi}\int_0^{2\pi} x\,dx$$

$$= \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} = \frac{1}{\cancel{2\pi}}\left(\frac{(2\pi)^{\cancel{2}}}{2}\right)$$

$$= \frac{2\pi}{2} = \pi$$

$$a_n = \frac{2}{T}\int f(x)\cos\frac{2\pi n}{T}x\,dx$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int x\cos\frac{\cancel{2\pi} n}{\cancel{2\pi}}x\,dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x\cos nx\,dx$$

+	x	$\cos nx$
−	1	$\frac{\sin nx}{n}$
+	0	$-\frac{\cos nx}{n^2}$

$$= \frac{1}{\pi}\left(x\frac{\sin nx}{n} + \frac{\cos nx}{n^2}\right)_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\left(\frac{2\pi \cancel{\sin 2\pi n}^{\,0}}{n} + \frac{\cancel{\cos 2\pi n}^{\,1}}{n^2}\right) - \left(\frac{\cancel{\cos 0}^{\,1}}{n^2}\right)\right]$$

$$= \frac{1}{\pi}\left(\frac{1}{n^2} - \frac{1}{n^2}\right) = 0$$

$$b_n = \frac{2}{T}\int f(x)\sin\frac{2\pi n}{T}x\,dx$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int x\sin\frac{\cancel{2\pi} n}{\cancel{2\pi}}x\,dx$$

	مشتق	انتگرال
$+$	x	$\sin nx$
$-$	1	$-\frac{\cos nx}{n}$
$+$	0	$-\frac{\sin nx}{n^2}$

$$= \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx$$

$$= \frac{1}{\pi}\left(-\frac{x\cos nx}{n} + \frac{\sin nx}{n^2}\right)\Bigg|_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\left(-\frac{2\pi \cancel{\cos 2\pi n}^{\,1}}{n} + \frac{\cancel{\sin 2\pi n}^{\,0}}{n^2}\right) - \left(\frac{\cancel{\sin 0}^{\,0}}{n^2}\right)\right]$$

$$= \frac{1}{\cancel{\pi}}\left(-\frac{2\cancel{\pi}}{n}\right) = -\frac{2}{n}$$

۲) سری فوریه تابع $f(x)=x^2$ را در بازه $(-\pi,\pi)$ بیابید.

$$\int_{-a}^{a} \text{زوج} = 2\int_{0}^{a}$$

$$T=\pi-(-\pi)=2\pi$$

$$a_0=\frac{1}{T}\int f(x)\,dx=\frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{x^2}_{\text{زوج}}\,dx$$

$$=\frac{1}{\cancel{2}\pi}\times\cancel{2}\int_{0}^{\pi} x^2\,dx=\frac{1}{\pi}\left[\frac{x^3}{3}\right]_{0}^{\pi}$$

$$=\frac{1}{\pi}\left(\frac{\pi^3}{3}\right)=\frac{\pi^2}{3}$$

$$a_n=\frac{2}{T}\int f(x)\cos\frac{2\pi n}{T}x\,dx$$

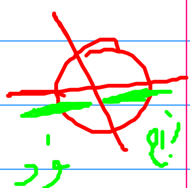
$$=\frac{\cancel{2}}{\cancel{2}\pi}\int_{-\pi}^{\pi} \underbrace{x^2}_{\substack{\text{زوج}\\ +}}\,\underbrace{\cos\frac{2\cancel{\pi} n}{2\cancel{\pi}}x}_{\substack{\text{زوج}\\ +}}\,dx$$

زوج

$$=\frac{1}{\pi}\times 2\int_{0}^{\pi} x^2\cos nx\,dx$$

	x^2	$\cos nx$
$+$	$2x$	$\frac{\sin nx}{n}$
$-$	2	$-\frac{\cos nx}{n^2}$
$+$	0	$-\frac{\sin nx}{n^3}$
$-$		

$$= \frac{2}{\pi}\left[x^2\frac{\sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2\sin nx}{n^3}\right]_0^{\pi}$$



$$= \frac{2}{\pi}\left[\left(\frac{\pi^2\cancelto{0}{\sin n\pi}}{n} + \frac{2\pi\overbrace{\cos n\pi}^{(-1)^n}}{n^2} - 2\,\frac{\cancelto{0}{\sin n\pi}}{n^3}\right) - \left(-2\,\frac{\cancelto{0}{\sin 0}}{n^3}\right)\right]$$

$$= \frac{2}{\cancel{\pi}}\left(\frac{2\cancel{\pi}(-1)^n}{n^2}\right) = \frac{4(-1)^n}{n^2}$$

$$b_n = \frac{2}{T}\int f(x)\sin\frac{2\pi n}{T}x\,dx$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int_{-\pi}^{\pi} \underbrace{x^2}_{\text{زوج}}\ \underbrace{\sin\frac{\cancel{2}\pi n}{\cancel{2}\pi}x}_{\text{فرد}}\,dx = 0$$

$+ \quad -$

$\underbrace{\qquad -\qquad}_{\text{فرد}}$

فرد زوج

۳) سری فوریه تابع $f(x)=\underbrace{x}_{\text{فرد}}\,|x|$ را در بازه $(-\pi,\pi)$ بیابید.

$$T=\pi-(-\pi)=2\pi$$

$$a_0=\frac{1}{T}\int f(x)\,dx=\frac{1}{2\pi}\int_{-\pi}^{\pi}\underbrace{x|x|}_{\text{فرد}}\,dx=0$$

$$a_n=\frac{2}{T}\int f(x)\cos\frac{2\pi n}{T}x\,dx$$

$$=\frac{2}{2\pi}\int_{-\pi}^{\pi}\underbrace{x|x|}_{\text{فرد}}\,\underbrace{\cos\frac{2\pi n}{2\pi}x}_{\text{زوج}}\,dx=0$$

فرد

$$b_n=\frac{2}{T}\int f(x)\sin\frac{2\pi n}{T}x\,dx$$

$$= \frac{\cancel{2}}{\cancel{2}\pi}\int_{-\pi}^{\pi} x|x| \sin \frac{\cancel{2}\pi n}{\cancel{2}\pi} x \, dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} \underbrace{x|x|}_{\text{فرد}} \, \underbrace{\sin nx}_{\text{فرد}} \, dx$$

$$\underbrace{- \quad\quad -}_{+}$$

زوج

$$= \frac{1}{\pi} \times 2 \int_{0}^{\pi} x \underset{x}{\underset{\|}{|x|}} \sin nx \, dx$$

$x > 0$

$$= \frac{1}{\pi} \times 2 \int_{0}^{\pi} x^2 \sin nx \, dx$$

$+$	x^2	$\sin nx$
$-$	$2x$	$-\frac{\cos nx}{n}$
$+$	2	$-\frac{\sin nx}{n^2}$
$-$	0	$\frac{\cos nx}{n^3}$

$$= \frac{1}{\pi} \times 2 \left[\frac{-x^2 \cos nx}{n} + \frac{2x \sin nx}{n^2} + \frac{2 \cos nx}{n^3} \right]_{0}^{\pi}$$

$$= \frac{2}{\pi}\left[\left(-\frac{\pi^2 \overset{(-1)^n}{\cos n\pi}}{n} + \frac{2\pi \overset{0}{\cancel{\sin n\pi}}}{n^2} + 2\,\frac{\overset{(-1)^n}{\cos n\pi}}{n^3} \right) \right.$$

$$- \left(\frac{r\, \cancel{\cos 0}^{\,1}}{n^2} \right) \Bigg]$$

$$= \frac{r}{\pi} \left(\frac{-\pi^{r} (-1)^n}{n} + \frac{r(-1)^n}{n^2} - \frac{r}{n^2} \right)$$