تمرین شماره دو

$\square + i\square$

حاصل عبارت‌های زیر را بدست آورید.

$$i^{2} = i \times i = (0-1) + i(0+0) = -1 + 0 = -1$$

$$i^{3} = i^{2} \times i = (-1)i = -i$$

$$i^{4} = i^{2} \times i^{2} = (-1)(-1) = 1$$

$$i^{95} = i^{4} \times i = (i^{2})^{32} \times i = (-1)^{32} \times i$$

$$= (+1)(i) = i$$

$$\frac{2+i}{3+4i} = \left(\frac{6+4}{3^2+4^2}\right) + \left(\frac{3-8}{3^2+4^2}\right)i$$

$$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \quad \frac{10}{25} + \frac{-5}{25}i$$

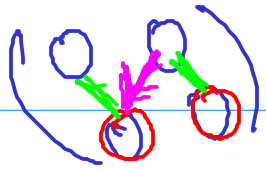
$$\mathrm{Im}\left(\frac{1}{1+i}\right) = -\frac{1}{2}$$

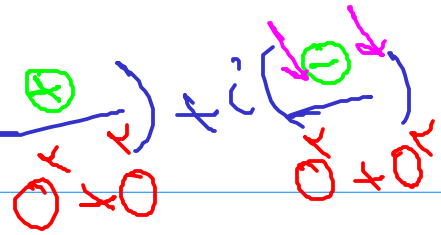
$$\frac{1}{1+i} = \left(\frac{1+0}{1^2+1^2}\right) + i\left(\frac{0-1}{1^2+1^2}\right)$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \frac{1}{2} + \left(-\frac{1}{2}\right)i$$

$$(-9+4i)(-6-i) = (54-(-4)) + i(-18+9)$$

$$\begin{pmatrix} -9 & 4 \\ -4 & -1 \end{pmatrix} = 58 - 9i$$





تمرین کلاسی ۳

حاصل عبارت‌های زیر را بدست آورید.

$$z^n = r^n e^{in\theta}$$

1) $(-1+i)^{14} = (\sqrt{2})^{14} e^{i(14)(\frac{3\pi}{4})}$

$-1+i$

$\begin{cases} x=-1 \\ y=1 \end{cases}$

$r = \sqrt{x^2+y^2} = \sqrt{1+1} = \sqrt{2}$

$tg\theta = \frac{y}{x} = \frac{1}{-1} = -1$

$\theta = \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

2) $i^i = 1^i e^{i(i)\frac{\pi}{2}} = e^{i^2\frac{\pi}{2}} = e^{-\frac{\pi}{2}}$ ($i^2 = -1$)

$z = r^n e^{in\theta}$

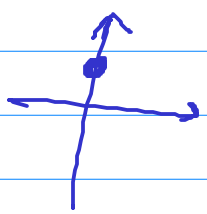
i^i

$x=0$

$y=1$

$r = \sqrt{0+1} = 1$

$tg\theta = \frac{1}{0} = \infty \Rightarrow \theta = \frac{\pi}{2}$



۳) $(-1-i\sqrt{3})^{-10} = 2^{-10}\, e^{i(-10)\frac{4\pi}{3}}$

$-1-i\sqrt{3}$

$\begin{cases} x=-1 \\ y=-\sqrt{3} \end{cases}$

$r=\sqrt{1+3}=2$

$\tan\theta = \frac{-\sqrt{3}}{-1} = \sqrt{3}$

$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

۴) $\left|\frac{2+i}{4+4i}\right| = \frac{\sqrt{5}}{5}$ $\frac{r_1}{r_2}$

$2+i$

$\begin{cases} x=2 \\ y=1 \end{cases}$

$r=\sqrt{4+1}=\sqrt{5}$

$3+4i$

$\begin{cases} x=3 \\ y=4 \end{cases}$

$r=\sqrt{9+16}=\sqrt{25}=5$

a) $\left|(-9+2i)(-7-i)\right| = \sqrt{90}\sqrt{25}$

$-9+2i$

$\begin{cases} x = -9 \\ y = 2 \end{cases}$ $\quad r = \sqrt{81+9} = \sqrt{90}$

$-4-i$

$x = -4$

$y = -1$ $\quad r = \sqrt{2^2+1} = \sqrt{25}$