تمرین شماره چهار

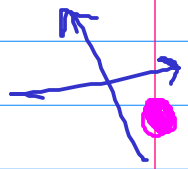
(1) جذر اعداد مختلط زیر را بیابید.

$$\sqrt[n]{z} = \sqrt[n]{r}\, e^{i\left(\frac{2k\pi+\theta}{n}\right)}$$

$$\sqrt[5]{-7-7i} = \sqrt[5]{\sqrt{98}}\, e^{i\left(\frac{2k\pi-\frac{\pi}{4}}{5}\right)}$$

$$k = 0, \dots, 4$$

$$-7-7i$$



$$\begin{cases} x = -7 \\ y = -7 \end{cases} \qquad r = \sqrt{49+49} = \sqrt{98}$$

$$\tan\theta = \frac{-7}{-7} = 1 \Rightarrow \theta = -\frac{3\pi}{4}$$

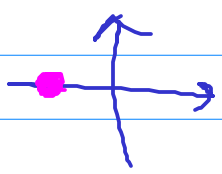
$$\sqrt[12]{-9} = \sqrt[12]{9}\, e^{i\left(\frac{2k\pi+\pi}{12}\right)}$$

$$k = 0, \dots, 11$$

$$-9+0i$$

$$\begin{cases} x = -9 \\ y = 0 \end{cases} \qquad r = \sqrt{81} = 9$$

$$\tan\theta = \frac{0}{-9} = 0 \Rightarrow \theta = \pi$$



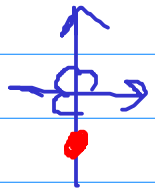
$$\sqrt[2]{-4i} = \sqrt{4}\, e^{i\left(\frac{2k\pi - \frac{\pi}{2}}{2}\right)}$$

$k = 0, 1$

$0 - 4i$

$\begin{cases} x = 0 \\ y = -4 \end{cases}$

$r = \sqrt{9} = 4$



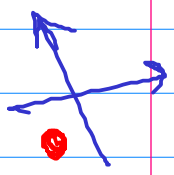
$\tan\theta = \frac{-4}{0} = \infty$

$\theta = -\frac{\pi}{2} = \frac{3\pi}{2}$

$$\sqrt[8]{-2 - 2\sqrt{3}i} = \sqrt{4}\, e^{i\left(\frac{2k\pi + \frac{4\pi}{3}}{8}\right)}$$

$k = 0, 1, \ldots, 7$

$-2 - 2\sqrt{3}i$



$\begin{cases} x = -2 \\ y = -2\sqrt{3} \end{cases}$

$r = \sqrt{4 + 12} = \sqrt{16} = 4$

$\tan\theta = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$

$\theta = \pi + \frac{\pi}{3}$

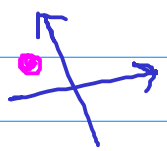
$= \frac{4\pi}{3}$

$(2\sqrt{3})^2 = 4 \times 3$

$$\sqrt[3]{-4+2\sqrt{3}\,i} = \sqrt[3]{\sqrt{16}}\; e^{i\left(\frac{2k\pi+\frac{2\pi}{3}}{3}\right)}$$

$$k = 0, 1, 2$$

$$-4+2\sqrt{3}\,i$$



$$\begin{cases} a = -4 \\ b = 2\sqrt{3} \end{cases}$$

$$r = \sqrt{12+4} = \sqrt{16}$$

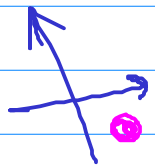
$$\tan\theta = \frac{b}{a} = \frac{2\sqrt{3}}{-4} = -\frac{\sqrt{3}}{2}$$

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\sqrt[4]{1-\sqrt{3}\,i} = \sqrt[4]{2}\; e^{i\left(\frac{2k\pi-\frac{\pi}{3}}{4}\right)}$$

$$k = 0, 1, \ldots, 3$$

$$1-\sqrt{3}\,i$$



$$\begin{cases} a = 1 \\ b = -\sqrt{3} \end{cases}$$

$$r = \sqrt{1+3} = 2$$

$$\tan\theta = -\sqrt{3} \Rightarrow \theta = -\frac{\pi}{3}$$

$z = x + iy$

آیا تابع $f(z) = 3z + 2$ کلی است؟ (2)

$f(x+iy) = 3(x+iy) + 2$ $\quad z \to w$

$= 3x + 3yi + 2$ $\quad x+iy \quad u+iv$

$= \underbrace{3x+2}_{u} + \underbrace{3y}_{v} i$

$\frac{\partial u}{\partial x} = 3$ $\quad \frac{\partial v}{\partial y} = 3$

$=$

$\frac{\partial u}{\partial y} = 0$ $\quad -\frac{\partial v}{\partial x} = 0$

$=$

شرایط کلی

$$\begin{cases} \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \end{cases}$$

$$f(z) = z^2 + 2$$

$$f(x+iy) = (x+iy)^2 + 2$$

$$= x^2 + (iy)^2 + 2x(iy) + 2$$

$$= x^2 + i^2 y^2 + 2xyi + 2$$

$$= x^2 - y^2 + 2xyi + 2$$

$$= \underbrace{x^2 - y^2 + 2}_{u} + \underbrace{2xy}_{v} i$$

$$\frac{\partial u}{\partial x} = 2x \qquad \frac{\partial v}{\partial y} = 2x$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -2y \qquad -\frac{\partial v}{\partial x} = -2y$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

كشي ريمان