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جلسه هفتم

تعریف: برای هر عدد مختلط $z = a + iy$ مزدوج \bar{z}

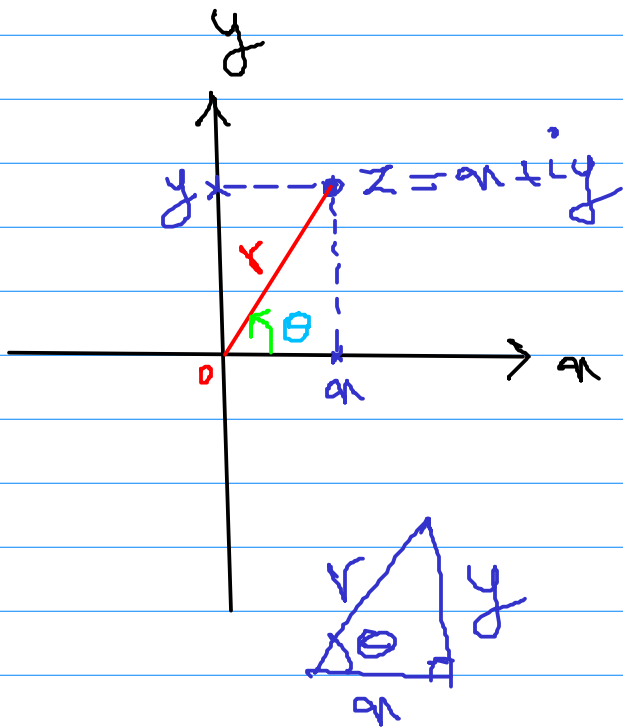
به صورت $\bar{z} = a - iy$ نمایش داده و به صورت زیر است

$$\bar{z} = a - iy$$

تعریف: قدر مطلق عدد مختلط $z = a + iy$ به صورت زیر
تعریف می شود

$$|z| = \sqrt{a^2 + y^2}$$

نمایش قطبی اعداد مختلط



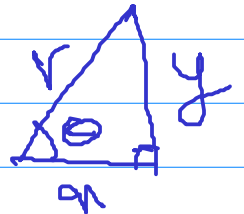
برای $z = a + iy$

$$\begin{cases} \sin \theta = \frac{y}{r} \Rightarrow y = r \sin \theta \\ \cos \theta = \frac{x}{r} \Rightarrow x = r \cos \theta \end{cases}$$

$$\begin{aligned} z = x + iy &= (r \cos \theta) + i(r \sin \theta) \\ &= r(\cos \theta + i \sin \theta) \\ &= r e^{i\theta} \end{aligned}$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\begin{cases} z = x + iy \\ z = r e^{i\theta} \end{cases}$$

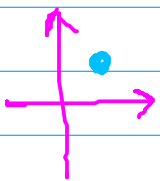


$$\begin{cases} r = \sqrt{x^2 + y^2} \\ \tan \theta = \frac{y}{x} \end{cases}$$

مثال: اعداد مختلط زیر را قطبی کنید.

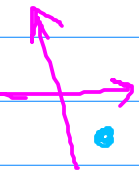
$$\begin{aligned} z &= 1 + i \\ &= \sqrt{2} e^{i\pi/4} \end{aligned}$$

$$\begin{aligned} \text{اول} \begin{cases} x = 1 \\ y = 1 \end{cases} & \quad r = \sqrt{x^2 + y^2} \\ & = \sqrt{1+1} = \sqrt{2} \end{aligned}$$



$$\operatorname{tg} \theta = \frac{y}{x} = \frac{1}{1} = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$z = 1 - i\sqrt{12} = \sqrt{13} e^{-\frac{12}{4}i}$$



$$\begin{cases} x = 1 \\ y = -\sqrt{12} \end{cases}$$

$r > 0$

$$r = \sqrt{x^2 + y^2}$$

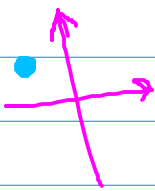
$$= \sqrt{1^2 + (-\sqrt{12})^2}$$

$$= \sqrt{1 + 12} = \sqrt{13}$$

$$\operatorname{tg} \theta = \frac{y}{x}$$

$$= \frac{-\sqrt{12}}{1} \Rightarrow \theta = -\frac{12}{4}$$

$$z = -1 + i\sqrt{12} = \sqrt{13} e^{\frac{12}{4}i}$$



$$\begin{cases} x = -1 \\ y = \sqrt{12} \end{cases}$$

$r > 0$

$$r = \sqrt{x^2 + y^2}$$

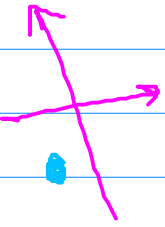
$$= \sqrt{(-1)^2 + (\sqrt{12})^2}$$

$$= \sqrt{1 + 12} = \sqrt{13}$$

$$\operatorname{tg} \theta = \frac{y}{x} = \frac{\sqrt{12}}{-1} = -\sqrt{12}$$

$$\theta = \pi - \frac{12}{4} = \frac{12}{4}$$

$$z = -r - ri = \sqrt{r} e^{\frac{3\pi}{4}i}$$



$$\begin{cases} x = -r \\ y = -r \end{cases}$$

راديان

$$r = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-r)^2 + (-r)^2}$$

$$= \sqrt{r + r} = \sqrt{r}$$

$$\arg z = \frac{y}{x} = \frac{-r}{-r} = 1$$

$$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$$

