جلسه نهم ۲۱/۲/۲

توان اعداد مختلط

اگر $Z=re^{i\theta}$ آنگاه $Z^n=r^n e^{in\theta}$

مثال:

$$(1+i)^{25}=\left(\sqrt{2}\right)^{25} e^{i(25)\frac{\pi}{4}}$$

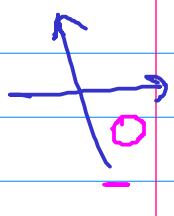
$1+i$

$$\begin{cases} x=1 \\ y=1 \end{cases}$$

$$r=\sqrt{1+1}=\sqrt{2}$$

$$\tan\theta=\frac{y}{x}=\frac{1}{1}=1 \Rightarrow \theta=\frac{\pi}{4}$$

$$(1-i)^9=\left(\sqrt{2}\right)^9 e^{i\times 9\left(-\frac{\pi}{4}\right)}$$



$$\begin{cases} x=1 \\ y=-1 \end{cases}$$

$$r=\sqrt{1+1}=\sqrt{2}$$

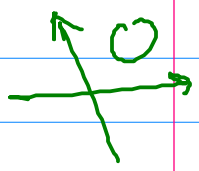
$$\tan\theta=\frac{y}{x}=\frac{-1}{1}=-1$$

$$\Rightarrow \theta=-\frac{\pi}{4}$$

$$\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{10} = \left(\frac{2e^{\frac{\pi}{3}i}}{2e^{-\frac{\pi}{3}i}}\right)^{10}$$

$$= \left(e^{\frac{\pi}{3}i-\left(-\frac{\pi}{3}i\right)}\right)^{10} = \left(e^{\frac{2\pi}{3}i}\right)^{10} = e^{\frac{20\pi}{3}i}$$

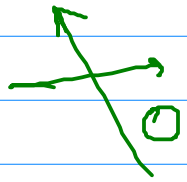
$$1+i\sqrt{3} =$$



$$\begin{cases} x=1 \\ y=\sqrt{3} \end{cases} \qquad r=\sqrt{1+3}=2 \qquad tg\theta=\frac{\sqrt{3}}{1}=\sqrt{3}$$

$$\theta=\frac{\pi}{3}$$

$$1-i\sqrt{3} =$$

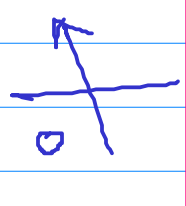


$$\begin{cases} x=1 \\ y=-\sqrt{3} \end{cases} \qquad r=\sqrt{1+3}=2 \qquad tg\theta=-\frac{\sqrt{3}}{1}$$

$$\theta=-\frac{\pi}{3}$$

$$\left(-3-i\sqrt{3}\right)^{5} = \left(\sqrt{12}\,e^{i(5)\left(\frac{4\pi}{6}\right)}\right)$$

$$-3-i\sqrt{3} \qquad r=\sqrt{9+3}=\sqrt{12}$$



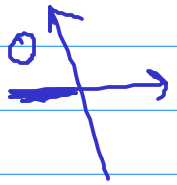
$$\begin{cases} x=-3 \\ y=-\sqrt{3} \end{cases} \qquad tg\theta=\frac{-\sqrt{3}}{-3}=\frac{\sqrt{3}}{3} \Rightarrow \theta=\pi+\frac{\pi}{6}$$

$$=\frac{4\pi}{6}$$

$$r^n e^{in\theta}$$

$$-\sqrt{\quad} \qquad -\sqrt{\quad}\; i(-\sqrt{\quad})\left(\frac{\pi}{\quad}\right)$$

$$(-\pi+\pi i) = (\sqrt{\quad})\; e$$



$$\begin{cases} x = -\pi \\ y = \pi \end{cases}$$

$$r = \sqrt{9+9} = \sqrt{18}$$

$$\tan\theta = \frac{y}{x} = \frac{\pi}{-\pi} = -1$$

$$\theta = \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

نلبث 5

$$z_1 = r_1 e^{i\theta_1} \qquad z_2 = r_2 e^{i\theta_2}$$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1+\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1-\theta_2)}$$

نلبث 5

$$z = re^{i\theta} \Rightarrow |z| = r$$

$$|z_1 z_2| = r_1 r_2 \qquad \left|\frac{z_1}{z_2}\right| = \frac{r_1}{r_2}$$

مثال ۸

$$\left|\frac{1-i}{\sqrt{3}+i}\right| = \frac{\sqrt{2}}{2}$$

$$1-i \longrightarrow r_1 = \sqrt{1+1} = \sqrt{2}$$

$$\sqrt{3}+i \longrightarrow r_2 = \sqrt{3+1} = 2$$

$$\left| i\,(-5+\sqrt{3}\,i)\right| = (1)(\sqrt{28}) = \sqrt{28}$$

$$i \Rightarrow 0+1i \qquad r_1 = \sqrt{0+1} = 1$$

$$-5+\sqrt{3}\,i \Longrightarrow r_2 = \sqrt{25+3} = \sqrt{28}$$

جذر اعداد مختلط

اگر $z = re^{i\theta}$ آنگاه

$$\sqrt[n]{z} = \sqrt[n]{r}\; e^{i\left(\frac{2k\pi+\theta}{n}\right)}$$

$$k = 0, 1, \ldots, n-1$$

مثال:

$$\sqrt[4]{-8i} = \sqrt[4]{8}\; e^{i\left(\frac{2k\pi - \frac{\pi}{2}}{4}\right)}$$

$$k = 0, 1, 2, 3$$

$-8i$

$$\begin{cases} 9R = 0 \\ y = -8 \end{cases}$$

$$r = \sqrt{0 + 64} = 8$$

$$\tan\theta = \frac{-8}{0} = \infty \Rightarrow \theta = -\frac{\pi}{2}$$